Exhibit 10

Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems

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2.1.1 Earth-Centered Inertial Frame

In physics, an *inertial* coordinate frame is one that does not accelerate or rotate with respect to the rest of the Universe. This does not define a unique coordinate frame. In navigation, a more specific form of the inertial frame, known as the *Earth-centered inertial* frame, is used, denoted by the symbol i. This is nominally centered at the Earth's center of mass and oriented with respect to the Earth's spin axis and the stars. Strictly, this is not a true inertial frame, as the Earth experiences acceleration in its orbit around the Sun, its spin axis slowly moves, and the Galaxy rotates. However, it is a sufficiently accurate approximation to an inertial frame for navigation purposes.[1]

Figure 2.2 shows the axes of the ECI frame. The rotation shown is that of the Earth with respect to space. The z-axis always points along the Earth's axis of rotation from the center to the north pole (true, not magnetic). The x- and y-axes lie within the equatorial plane. They do not rotate with the Earth, but the y-axis always lies 90 degrees ahead of the x-axis in the direction of rotation. This does not uniquely define the coordinate frame; it is also necessary to specify the time at which the inertial frame axes coincide with those of the ECEF frame (see Section 2.1.2).[2] There are two common solutions. The first is simply to align the two coordinate frames when the navigation solution is initialized. The other option, used within the scientific community, is to define the x-axis as the direction from the Earth to the Sun at the vernal equinox, which is the spring equinox in the northern hemisphere. This is the same as the direction from the center of the Earth to the intersection of the Earth's equatorial plane with the Earth-Sun orbital plane (ecliptic).

A further problem, where a precise definition of the coordinate frame is needed, is polar motion. The spin axis actually moves with respect to the solid Earth, with the poles roughly following a circular path of radius 15m. One solution is to adopt the IERS reference pole (IRP) or conventional terrestrial pole (CTP), which is the average position of the pole surveyed between 1900 and 1905. The version of the inertial frame that adopts the IRP/CTP, the Earth's center of mass as its origin, and the x-axis based on the Earth-Sun axis at vernal equinox is known as the conventional inertial reference system (CIRS).

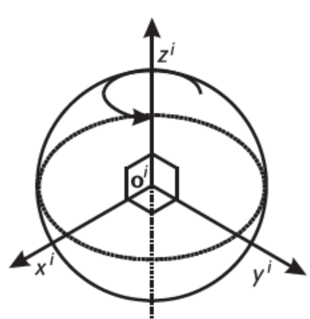

Figure 2.2 Axes of the ECI frame. (*From:* [1]. © 2002 QinetiQ Ltd. Reprinted with permission.)

The inertial frame is important in navigation because inertial sensors measure motion with respect to a generic inertial frame, and it enables the simplest form of navigation equations to be used, as shown in later chapters.

2.1.2 Earth-Centered Earth-Fixed Frame

The *Earth-centered Earth-fixed* frame, commonly abbreviated to Earth frame, is similar to the ECI frame, except that all axes remain fixed with respect to the Earth. The ECEF frame is denoted by the symbol e and has its origin at the center of the ellipsoid modeling the Earth's surface, which is roughly at the center of mass.

Figure 2.3 shows the axes of the ECEF frame. The z-axis always points along the Earth's axis of rotation from the center to the North Pole (true, not magnetic). The x-axis points from the center to the intersection of the equator with the IERS reference meridian (IRM) or conventional zero meridian (CZM), which defines 0 degree longitude. The y-axis completes the right-handed orthogonal set, pointing from the center to the intersection of the equator with the 90-degree east meridian. The ECEF frame using the IRP/CTP and the IRM/CZM is also known as the conventional terrestrial reference system (CTRS), and some authors use the symbol t to denote it.

The Earth frame is important in navigation because the user wants to know their position relative to the Earth, so it is commonly used as both a reference frame and a resolving frame.

2.1.3 Local Navigation Frame

The *local navigation frame*, local level navigation frame, geodetic, or geographic frame is denoted by the symbol n (some authors use g). Its origin is the point a navigation solution is sought for (i.e., the navigation system, the user, or the host vehicle's center of mass).[1]

Figure 2.4 shows the axes of the local navigation frame. The z axis, also known as the down (D) axis, is defined as the normal to the surface of the reference ellipsoid (Section 2.3.1), pointing roughly toward the center of the Earth. Simple gravity models (see Section 2.3.5) assume that the gravity vector is coincident with

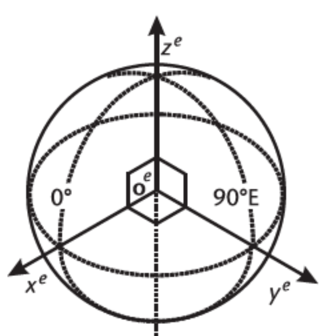

Figure 2.3 Axes of the ECEF frame. (*From:* [1]. © 2002 QinetiQ Ltd. Reprinted with permission.)

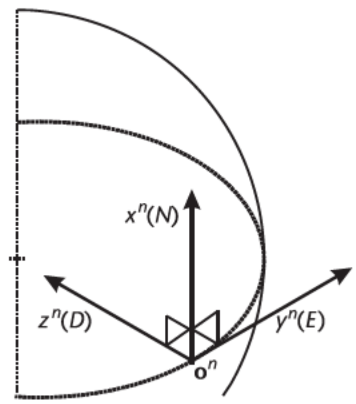

Figure 2.4 Axes of the local navigation frame. (*From:* [1]. © 2002 QinetiQ Ltd. Reprinted with permission.)

the z axis of the local navigation frame. True gravity deviates from this slightly due to local anomalies. The x-axis, or north (N) axis, is the projection in the plane orthogonal to the z-axis of the line from the user to the north pole. By completing the orthogonal set, the y-axis always points east and is hence known as the east (E) axis.

North, east, down is the most common form of the local navigation frame and will always be used here. However, there are other forms in use, such as x = east, y = north, z = up, and x = south, y = west, z = down.[2]

The local navigation frame is important in navigation because the user wants to know their attitude relative to the north, east, and down directions. For position and velocity, it provides a convenient set of resolving axes, but is not used as a reference frame.

A major drawback of the local navigation frame is that there is a singularity at each pole because the north and east axes are undefined there. Thus, navigation equations mechanized using this frame are unsuitable for use near the poles. Instead, an alternative frame should be used with conversion of the navigation solution to the local navigation frame at the end of the processing chain.

In a multibody problem, there are several local navigation frames in play. However, in practice, only one tends to be of interest. Also, the differences in orientation between the local navigation frames of objects within a few meters can usually be neglected.[3]

2.1.4 Body Frame

The *body frame*, sometimes known as the vehicle frame, comprises the origin and orientation of the object for which a navigation solution is sought. The origin is coincident with that of the local navigation frame, but the axes remain fixed with respect to the body and are generally defined as x = forward (i.e., the usual direction of travel), z = down (i.e., the usual direction of gravity), and y = right, completing the orthogonal set. For angular motion, the x-axis is the roll axis, the y-axis is the pitch axis, and the z-axis is the yaw axis. Hence, the axes of the body frame are sometimes known as roll, pitch, and yaw. Figure 2.5 illustrates this. A right-handed

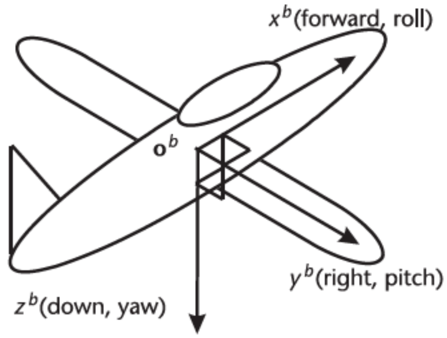

Figure 2.5 Body frame axes. (*From:* [1]. © 2002 QinetiQ Ltd. Reprinted with permission.)

corkscrew rule applies, whereby if the axis is pointing away, then positive rotation about that axis is clockwise.[1]

The body frame is essential in navigation because it describes the object that is navigating. All strapdown inertial sensors measure the motion of the body frame (with respect to a generic inertial frame).

The symbol b is used to denote the body frame of the primary object of interest. However, many navigation problems involve multiple objects, each with their own body frame, for which alternative symbols, such as s for a satellite and a for an antenna, must be used.

2.1.5 Other Frames

The *geocentric frame*, denoted c, is similar to the local navigation frame, except that the z axis points from the origin to the center of the Earth. The x-axis is again the projection of the line to the north pole in the orthogonal plane to the z-axis.[1]

The *tangent plane frame*, t, is also known as the local geodetic frame. It is similar to the local navigation frame, except that the origin does not coincide with the navigating object but is instead fixed relative to the Earth. This frame is used for navigation within a localized area, such as aircraft landing.

The origin and z-axis of the *wander azimuth frame*, w (some authors use n), are coincident with that of the local navigation frame. However, the x and y axes are displaced from north and east by an angle, ψ_{nw} or α, known as the wander angle, that varies as the frame moves with respect to the Earth. Use of this frame avoids the polar singularity of the local navigation frame, so it is commonly used to mechanize inertial navigation equations. The wander angle is always known, so transformation of the navigation solution to the local navigation frame is straightforward. Wander-azimuth navigation equations are summarized in Section 5.3.5.

In most systems, the sensitive axes of the inertial instruments are nominally aligned with the body frame axes, but there will always be some deviation. Therefore, some authors like to treat these sensitive axes as separate *inertial instrument frames*, with a coordinate frame for each accelerometer and gyro. However, it is generally simpler to treat the departures from the body axes (i.e., the instrument mounting misalignments), as a set of perturbations. Some IMUs are manufactured in a "skew" configuration, whereby the sensors' sensitive axes are not aligned with